ANDHRA UNIVERSITY
TRANS-DISCIPLINARY RESEARCH HUB

TOPICS IN NUMBER THEORY

Unit-I

Divisibility Theory in the Integers: The Divisibility Algorithm, The Greatest Common Divisor, The Euclidean Algorithm, The Diophantine Equation $ax + by = c$.

Primes and Their Distribution: The Fundamental theorem of arithmetic, The sieve of Eratosthenes, The Goldbach conjecture.

(Sections 2.1 to 2.4 of Chapter-2 and Sections 3.1 to 3.3 of Chapter-3 in the prescribed text book)

Unit-II

The theory of Congruences: Karl Friedrich Gauss, Basic properties of Congruence, special divisibility tests, Linear congruences.

Fermats theorem: Pierre de Fermat, fermat factorization method, the little theorem, Wilson's theorem.

(Section 4.1 to 4.4 of Chapter-4 and Section 5.1 to 5.4 of Chapter-5 in the prescribed text book)

Unit-III

Number-theoritic functions: The functions τ and σ, The Mobius Inversion formula, The greatest Integer function.

Perfect numbers: The search for Perfect numbers, Mersenne primes, Fermat numbers.

(Section 6.1 to 6.3 of Chapter-6 and Section 10.1 to 10.3 of Chapter 10 in the prescribed text book)

Unit-IV

Fibonacci numbers and Continued fractions: The Fibonacci sequence,
Certain identities involving Fibonacci numbers, Finite Continued fractions, Infinite Continued fractions, Pell's equation.

(Section 13.1 to 13.5 of Chapter-13 in the prescribed text book)

Prescribed Text book: Elementary Number Theory, Second Edition by David M. Burton

References:

1. An introduction to the theory of numbers, Fifth edition by Ivan Niven, Herberts Zuckerman and Hugh L. Montgometry,
2. Number Theory by George E. Andrews.

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Time: 3 hours | Model paper | Maximum marks: 100

Answer any five of the following questions.

All questions carry equal marks.

1. (a) Show that the linear Diophantine equation $ax+by=c$ has a solution if and only if d/c, where $d= \gcd(a,b)$. If x_0, y_0 is any particular solution of this equation, then all the other solutions are given by $x=x_0 + (b/d)t$, $y=y_0 – (a/d)t$ for varying integers t.

   (b)Determine all solutions in the integers of the following Diophantine equation

   $56x + 72y = 40$.

2. (a) Show that there are an infinite number of primes.
   (b) Show that the Goldbach's conjecture statement that every even integer greater than 2 is the sum of two primes is equivalent to the statement that every integer greater than 5 is the sum of three primes.

3. (a) Solve the linear congruence $9x\equiv 21 \pmod{30}$.
   (b) Solve the Diophantine equation $4x+51y = 9$ using congruences.

4. (a) If p and q are distinct primes such that $a^p\equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$, then show that $a^{pq} \equiv a \pmod{pq}$.
   (b) show that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod{4}$.

5. If $n = p_1^{k_1}p_2^{k_2} \ldots. p_r^{k_r}$is the prime factorization of n.>1, then

   (a) $\tau(n) = (k_1 + 1)(k_2 + 1)\ldots.(k_r+ 1)$, and

   (b) $\sigma(n) = \dfrac{p_1^{k_1+1} – 1}{p_1 – 1} \dfrac{p_2^{k_2+1} – 1}{p_2 – 1} \ldots.. \dfrac{p_r^{k_r+1} – 1}{p_r – 1}$.

6. (a) If 2^k- 1 is prime (k>1) then show that n = $2^{k-1}(2^k - 1)$ is perfect and every even perfect number is of this form.

(b) Show that for n ≥ 1 the Fermat number $F_n = 2^{2n} + 1$ is prime if and only if $3^{(F_n - 1)/2} \equiv -1 \pmod{F_n}$.

7. (a) Show that the greatest common divisor of two Fibonacci numbers is again a Fibonacci number.

(b) Prove that there is no positive integer n for which

$u_1 + u_2 + u_3 + \ldots + u_{3n} = 16!$.

8. (a) Solve the linear Diophantine equation 172x+20y = 1000.

(b) Show that the value of any infinite continued fraction is an irrational number.